

FIXED CHARGE CAPACITATED TRANSPORTATION PROBLEM UNDER VEHICLES

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Abstract

The aim of this paper is to develop a methodology to solve fixed charge capacitated transportation problem (FCCTP) under finite vehicles. In this paper we proposed not only the mathematical model or problem and solution methodology of transportation problem under finite vehicles, but method of IBFS, determination of unit transportation costs. Bi-level mathematical programming model is formulated for cost varying transportation problem (CVTP). The bi-level mathematical programming for Cost Varying Fixed Charge Capacitated Transportation Problem (CVFCCTP) is developed. We solve this model by standard software. A Numerical example is presented to illustrate CVFCCTP.

Keywords

Transportation Problem, Cost Varying Transportation Problem, Fixed Charge Transportation Problem, Capacitated Transportation Problem, Bi-level Mathematical Programming.

Introduction

Transportation problem(TP) is a special class of linear programming problem. This problem deals with the distribution of single commodity from various sources of supply to various destinations of demand in such a manner that the total transportation cost is minimized. For this problem the following information are to be needed:

(P1) Available amount of the commodity at different origins.

(P2) Amounts demanded at different destinations.

(P3) The transportation cost of one unit of commodity from various origin to various destination.

The unit transportation cost is not constant. The ambiguity is found in pre-paid transportation system in our daily life. To transport quantities we contract a vehicle in a fixed charge for a single trip. Different types of vehicles should be contracted in different charges depending on the carrying capacity. So depending on the vehicles the unit transportation cost is varied. From this angle of view, (P3) is changed as the transportation cost of unit

quantity is not constant where as the cost of single trip of each vehicle is constant which depends on its carrying capacity. This type of TP is named as CVTP.

In TP it is seen that the solution of TP is depended on its initial basic feasible solution (IBFS) and optimality test. In TP, the IBFS are determined by any one of North West Corner Method (NWCM) or Row Minimum Method (RMM) or Column Minimum Method (CMM) or Matrix Minimum Method (MMM) or Vogel's Approximation Method (VAM). The methods (except NWCM) are fully dependent on unit transportation cost. To determine the IBFS for CVTP Panda and Das [7, 8, 9, 10, 11, 12, 13] have used NWCM. In this paper, other techniques have to be described to determine IBFS for CVTP. The modification of the methods are described by fixed charges of vehicles. Since optimality test depends on unit transportation cost, so unit transportation cost for CVTP are determined by Das and Panda's [7, 8, 9, 10, 11, 12, 13] proposed algorithm. It is noted that the unit transportation unit costs may be changed during optimality test.

The fixed charge transportation problem (FCTP) is an extension of classical transportation which is fixed cost incurred for every origin. The fixed charge transportation problem was originally formulated by Hirsch and Danzig [3]. Many distribution problems in practice can only be formulated as FCTPs. The fixed cost may represent the cost of renting a vehicle, landing fees at airport, set up costs for machines in manufacturing environment etc. Capacitated transportation problem (CTP) is such type of TP where some route restrictions are given from all source to all destinations. In this paper we consider both together the situations FCTP as well as CTP and solve new problem by considering finite number of vehicles.

2 Mathematical Formulation

2.1 Fixed Charge Transportation Problem

The FCTP can be formulated mathematically as:

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} + \sum_{i=1}^m F_i \quad (1)$$

$$\text{Subject to } \sum_{j=1}^n x_{ij} = a_i \quad a_i > 0, i = 1, 2, 3, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j \quad b_j > 0, j = 1, 2, 3, \dots, n$$

$$x_{ij} \geq 0 \quad \forall i, j \quad (2)$$

Where, $i = 1, 2, 3, \dots, m$ is the set of origins.

$j = 1, 2, 3, \dots, n$ is the set of destinations.

x_{ij} = the quantity transported from the i^{th} origin to the j^{th} destination.

c_{ij} = per unit cost in transporting goods from the i^{th} origin to the j^{th} destination.

a_i = the amount available at the i^{th} origin.

b_j = the demand of the j^{th} destination.

F_i = the fixed cost associated with origin i .

2.2 Capacitated transportation problem:

Consider m origins and n destinations. At each origin O_i , let a_i be the amount of a homogeneous product which we want to transport to n destinations D_j to satisfy the demand for b_j units of the product there. The unit transportation cost is c_{ij} from source i to destination j . A variable x_{ij} represents the unknown quantity to transported from origin O_i to destination D_j . Let r_{ij} be the capacitated restrictions on route i, j for capacitated transportation problem.

A capacitated transportation problem can be stated in **Model 1** as follows:

Model 1

$$\min \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij},$$

$$\text{subject to } \sum_{j=1}^n x_{ij} = a_i, \quad i = 1, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = 1, \dots, n$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j \quad (3)$$

$$0 \leq x_{ij} \leq r_{ij} \quad \forall i, \forall j$$

The condition (3) is necessary condition for the **Model 1** to have a feasible solution, however, this is not sufficient because of the capacity restriction on each route.

To solve **Model 1** the stepwise procedure as follows:

Step 1:

Select vehicles V_1, V_2, \dots, V_N such that the capacities $C_K, K=1, \dots, N$ where $(C_1 < C_2 < \dots < C_N)$ of the vehicles $V_1 \dots V_2$ respectively, satisfies following restrictions.

$$C_K \leq r_{ij} \quad \forall i, j \forall K \quad (4)$$

so that **Model 1** converted to a N -vehicle cost varying transportation problem. For example, Consider

Example

Consider a capacitated transportation problem as

are as

	D_1	D_2	D_3	stock
O_1	.85	1.7	1.5	75
O_2	1.4	2.95	2.9	60
O_3	0.4	1.3	1.7	40
Demand	50	55	70	

with capacity of route restrictions

$r_{11} = 32, r_{12} = 30, r_{13} = 37;$
 $r_{21} = 35, r_{22} = 22, r_{23} = 38;$

$$r_{31} = 22, r_{32} = 36, r_{33} = 30;$$

Select the three vehicles where, the capacities of vehicles of V_1, V_2 and V_3 are respectively, $C_1 = 10, C_2 = 15$ and $C_3 = 20$. So that the route restriction is violated. Also, the

fixed cost for each vehicles are shown in each cell (i, j) in following tabulated form of 3-vehicle cost varying transportation problem.

	D_1	D_2	D_3	stock
O_1	5,8,10	10,13,15	7,10,12	75
O_2	4,7,9	6,9,11	8,11,13	60
O_3	12,15,17	3,6,8	5,8,10	40
Demand	50	55	70	

2.3 Cost Varying Transportation Problem(CVTP)

Suppose there are N -types off vehicles $V_r, r=1, \dots, N$ from each source to each destination. Let $C_r, r=1, \dots, N$ are the capacities(in unit) of the vehicles $V_r, r=1, \dots, N$ respectively, where $C_1 < C_2 < \dots < C_N$. So, CVTP can be represent in the following tabulated form.

	D_1	D_2	..	D_n	stock
O_1	$R_{11}^1, \dots, R_{11}^N$	$R_{12}^1, \dots, R_{12}^N$	$R_{1n}^1, \dots, R_{1n}^N$	a_1
O_2	$R_{21}^1, \dots, R_{21}^N$	$R_{22}^1, \dots, R_{22}^N$	$R_{2n}^1, \dots, R_{2n}^N$	a_2
....
O_m	$R_{m1}^1, \dots, R_{m1}^N$	$R_{m2}^1, \dots, R_{m2}^N$	$R_{mn}^1, \dots, R_{mn}^N$	a_m
Demand	b_1	b_2	b_n	

Table-1: Tabular representation of CVTP under N -vehicle

where a_i is the quantity of material available at source $O_i, i = 1, \dots, m$

b_j is the quantity of material required at destination $D_j, j = 1, \dots, n$

$R_{ij} = (R_{ij}^1, \dots, R_{ij}^N)$ represents transportation cost for each cell (i, j) ; where R_{ij}^l is the transportation cost from source $O_i, i = 1, \dots, m$ to the destination $D_j, j = 1, \dots, n$ by the vehicle $V_l, l = 1, \dots, N$. c_{ij} is the unit cost of transportation from st source O_i to destination D_j .

Definition 1: Feasible Solution (FS):

A set of non-negative allocations $x_{ij} \geq 0$ which satisfies (1),(2) is known as feasible solution.

Definition 2: Basic Feasible Solution (BFS):

A feasible solution to a m -origin and n -destination problem is said to be basic feasible solution if number of positive allocations are $(m+n-1)$.

If the number of allocations in a basic feasible solutions are less than $(m+n-1)$, it is called degenerate basic feasible solution (DBFS) otherwise non-degenerate basic feasible solution (NDBFS).

Definition 3: Optimal Solution:

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

2.3.1 Solution Method CVTP

The transportation methods for finding an optimum solution to the T.P. consists of the following steps:

- Find an initial basic feasible solution (IBFS)
- Test the optimality of IBFS.
- If the solution is optimal, end the process. Otherwise, improve it by a 'jump' to an adjacent basic feasible solution that yields the highest rate of improvement in the value of the objective function.
- Return to step (ii) and repeat the process until an optimal solution has been obtain.

2.3.2 Determination of IBFS

To determine the IBFS we apply any one of the following procedure

North-West corner Method (NWCM)

Step 1. Compute $\min(a_1, b_1)$. If $a_1 < b_1$, $\min(a_1, b_1) = a_1$ and if $a_1 > b_1$, $\min(a_1, b_1) = b_1$.

Select $x_{11} = \min(a_1, b_1)$ allocate the value of x_{11} in the cell (1,1).

Step 2. If $a_1 < b_1$, compute $\min(a_2, b_1 - a_1)$. Select $x_{21} = \min(a_2, b_1 - a_1)$ and allocate the

value of x_{21} in the cell (2,1).

If $a_1 > b_1$, compute $\min(a_1 - b_1, b_2)$. Select $x_{12} = \min(a_1 - b_1, b_2)$ and allocate the

value of x_{12} in the cell (1,2).

Let us now make an assumption that $a_1 - b_1 < b_2$. With this assumption the next cell

for which some allocation is to made, is the cell (2,2).

If $a_1 = b_1$ then allocate 0 only in one of two cells (2,1) or (1,2). The next allocation

is to be made cell (2,2).

In general, if an allocation is made in the cell $(i+1, j)$ in the current step, the next

allocation will be made either in cell (i, j) or $(i, j+1)$.

The feasible solution obtained by this away is always a BFS.

Example 1

Consider a CVTP as

	D_1	D_2	D_3	stock
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O_1	10,13	14,20	12,17	30
O_2	6,8	8,12	7,10	20
O_3	20,24	25,30	22,27	50
<i>Demand</i>	40	30	30	

The capacities of vehicles of V_1 and V_2 are respectively, $C_1 = 15$ and $C_2 = 25$.

By NWCM, the IBFS **Example 1** is given as follows.

	D_1	D_2	D_3	<i>stock</i>
O_1	$x_{11} = 30$ 10,13	14,20	12,17	30
O_2	$x_{21} = 10$ 6,8	$x_{22} = 10$ 8,12	7,10	20
O_3	20,24	$x_{32} = 20$ 25,30	$x_{33} = 30$ 22,27	50
<i>Demand</i>	40	30	30	

Modified row-minima method(MRMM)

In this method, we first consider the first row and find the minimum cost cell. Let $(1, k)$ cell be the cell in the first row with minimum cost entities $(R_{l,k}^1, R_{l,k}^2)$. We allot in this cell the maximum allocation, i.e, $x_{1k} = \min(a_1, b_k)$. If $a_1 < b_k$, then $x_{1k} = a_1$ and we cross out

the first row and consider the remaining tableau and proceed in same way. Again if $a_1 > b_k$, then $x_{1k} = b_k$ and we cross out the k^{th} column and consider the remaining row of the tableau and proceed next in the same way. If $a_1 = b_1$, then either 1st row or the k^{th} column will be crossed out and the remaining tableau will be consider.

By MRMM, the IBFS **Example 1** is given as follows.

	D_1	D_2	D_3	stock
O_1	$x_{11} = 30$ 10,13	14,20	12,17	30
O_2	$x_{21} = 10$ 6,8	8,12	$x_{23} = 10$ 7,10	20
O_3	20,24	$x_{32} = 30$ 25,30	$x_{33} = 20$ 22,27	50
<i>Deman</i> <i>d</i>	40	30	30	

Modified column-minima method(MCMM)

This method is exactly same as the Row-minima method. In this method, we are to start with first column instead of first row and the successive steps we consider only columns.

By MCMM, the IBFS of **Example 1** is given as follows.

	D_1	D_2	D_3	stock
O_1	$x_{11} = 20$ 10,13	$x_{12} = 10$ 14,20	12,17	30
O_2	$x_{21} = 20$ 6,8	8,12	7,10	20
O_3	20,24	$x_{32} = 20$ 25,30	$x_{33} = 30$ 22,27	50
<i>Demand</i>	40	30	30	

Modified matrix-minima method(MMMM)

This method finds a better starting solution. In this method we first find out the cell with minimum cost entities in the cost matrix and allocate in that cell the maximum allowable amount. We then cross out the satisfied row or column and adjust the amounts of supply and demand accordingly. We repeat the process with uncrossed out matrix and we are left at the end with exactly one uncrossed out row or column. If the cell with minimum cost is not unique, then any one of these cells may be selected for allotment.

By MCMM, the IBFS of **Example 1** is given as follows.

	D_1	D_2	D_3	stock
O_1	$x_{11} = 20$ 10,13	14,20	$x_{13} = 10$ 12,17	30
O_2	$x_{21} = 20$ 6,8	8,12	7,10	20
O_3		$x_{32} = 30$	$x_{33} = 20$	50

	20,24	25,30	22,27	
<i>Demand</i>	40	30	30	

Modified Vogel’s approximation Method(MVAM)

In this method the allocation is made on the basis of the opportunity (or penalty orextra) cost entities that would be incurred if allocation in certain cells with minimum unit transportation cost entities were missed. The steps in modified Vogel’s approximation method(MVAM) are as follows:

Step 1. Calculate the penalties for each row(column) by taking the differences between the smallest and

next smallest transportation cost entities in the same row (column) and write them in brackets against the corresponding row (column).

Step 2. Select the row or column with the largest penalty entities. If there is a tie in the values of penalties entities, then it can be broken by selecting the cell where the maximum allocation can be made.

Step 3. Allocate as much as possible in the lowest cost entities of the row(or column)which is defined by the Step 2.

Step 4. Adjust the supply and demand and cross-out the satisfied row or column.

Step 5. Repeat Step 1. Step 2. until the entire available supply at various sources and demand at various destinations are fully satisfied.

By **MVAM**, the IBFS of Example 2.1 is given as follows.

	D_1	D_2	D_3	<i>stock</i>
O_1	$x_{11} = 30$ 10,13	14,20	12,17	30
O_2		$x_{22} = 20$		20

	6,8	8,12	7,10	
O_3	$x_{31} = 10$ 20,24	$x_{32} = 10$ 25,30	$x_{33} = 30$ 22,27	50
<i>Deman</i> <i>d</i>	40	30	30	

2.3.3 Determination of C_{ij} in CVTP

To solve this problem, apply our proposed algorithms stated as follows:

2.3.4 Algorithm

Step 1. Determine IBFS by any of NWCM, MRMM, MCM, MMM, MVAM.

Step 2. After the allocate x_{ij} , determine c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{\sum t_r R_{ij}^r}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

where t_r are integer solution of $\min \sum t_r R_{ij}^r$

$$s.t. x_{ij} \leq \sum t_r C_r$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij} , then for non-basic cell c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{\sum t_r R_{ij}^r}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

where t_r are integer solution of $\min \sum t_r R_{ij}^r$

$$s.t. x_{ij} \leq \sum t_r C_r$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix c_{ij} by **Step 2** and for non-basic we fix c_{ij} by **Step 3**.

Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

Using Algorithm(TP1), in **Example 2.1**, the unit transportation costs in basic cells (by MVAM) are $c_{11} = \frac{39}{30}$, $c_{22} = \frac{24}{20}$, $c_{31} = \frac{24}{10}$, $c_{32} = \frac{30}{10}$, $c_{33} = \frac{31}{10}$

and the unit transportation costs in non-basic cells (by MVAM) are

$$c_{12} = \frac{20}{10}, c_{13} = \frac{51}{30}, c_{21} = \frac{9}{10}, c_{23} = \frac{20}{20}$$

2.3.3 Optimality test:

Once an initial basic feasible solution has been computed, the next step in the transportation problem is to determine whether the solution obtained is optimum or not. The method for testing optimality by cell evaluation is $u-v$ method.

$u-v$ method:

In order to test for optimality we should follow the $u-v$ method which is given below:

Step 1. Start with B.F.S. consisting of $m+n-1$ allocation in independent positions.

Step 2. Determine a set of $m+n$ numbers $u_i, i=1, \dots, m$ and $v_j, j=1, \dots, n$ such that in each cell (i, j) $c_{ij} = u_i + v_j$

Step 3. Calculate cell evaluations (unit cost difference) d_{ij} for each empty cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

Step 4. Examine the matrix of cell evaluation d_{ij} for negative entries and conclude that

- (i) If all $d_{ij} > 0$, then Solution is optimal and unique.
- (ii) If all $d_{ij} \geq 0$ and at least one $d_{ij} = 0$, then solution is optimal and alternative solution also exists.
- (iii) If at least one $d_{ij} < 0$, then solution is not optimal.

If it is so, further improvement is required by repeating the above process after Step 5 and onwards.

Step 5. (i) See the most negative cell in the matrix $[d_{ij}]$.

(ii) Allocate θ to this empty cell in the final allocation table. Subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.

(iii) This value of θ , in general is obtained by equating to zero the minimum of the allocations containing $-\theta$ (not $+\theta$) only at the corners of the closed loop.

(iv) Substitute the value of θ and find a fresh allocation table.

Step 6. Again, apply the above test for optimality till we find $d_{ij} \geq 0$.

Optimality test of Example 2.1 after MVAM and c_{ij} Determine a set of 6 numbers $u_i, i=1,2,3$ and $v_j, j=1,2,3$ such that in each cell basic (i, j) $c_{ij} = u_i + v_j$,

each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i=1,2,3$, $v_j, j=1,2,3$ and d_{ij} non-basic cell (i, j) is given in the following table

	D_1	D_2	D_3	stock
O_1	$x_{11} = 30$ $c_{11} = \frac{39}{30}$ 10,13	$d_{12} > 0$ $c_{12} = \frac{20}{10}$ 14,20	$d_{13} > 0$ $c_{13} = \frac{51}{30}$ 12,17	30
O_2	$d_{21} > 0$ $c_{21} = \frac{9}{10}$ 6,8	$x_{22} = 20$ $c_{22} = \frac{24}{20}$ 8,12	$d_{23} > 0$ $c_{23} = \frac{20}{20}$ 7,10	20
O_3	$x_{31} = 10$ $c_{31} = \frac{24}{10}$ 20,24	$x_{32} = 10$ $c_{32} = \frac{30}{10}$ 25,30	$x_{33} = 30$ $c_{33} = \frac{31}{10}$ 22,27	50
Demand	40	30	30	

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution.
 $x_{11} = 30, x_{22} = 20, x_{31} = 10, x_{32} = 10, x_{33} = 31$. Minimum cost $Z^* = 30 + 20 + 10 + 10 + 31 = 101$ unit(Rs.)

The mathematical programming for CVTP under N vehicle is described below.

2.3.4 Bi-level Mathematical Programming for CVTP under N -Vehicle

The Bi-level mathematical programming for CVTP under N -vehicle is formulated in **Model 2 (CVTPNV)** as follows:

Model 2 (CVTPNV)

$$\min \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

where, c_{ij} is determined by following

mathematical programming

$$c_{ij} = \begin{cases} \frac{\sum_r t_r R_{ij}^r}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

where $t_r, r = 1, \dots, N$

are integer solution of

$$\min \sum_r t_r R_{ij}^r$$

$$\text{s.t. } x_{ij} \leq \sum_r t_r C_r$$

$$\sum_{j=1}^n x_{ij} = a_i, \quad i = 1, \dots, m;$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = 1, \dots, n;$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j \tag{5}$$

$$x_{ij} \geq 0 \quad \forall i, \forall j$$

Theorem 2.1 The number of basic variables in a balanced CVTP is at most $(m+n-1)$.

Theorem 2.2 A necessary and sufficient condition for the existence of a feasible solution to a CVTP is

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

Theorem 2.3 The solution of a CVTP is never unbounded.

Theorem 2.4 A set X of column vectors of coefficient matrix of a CVTP will be linearly dependent if their corresponding cells in the transportation tableau contain a loop.

2.4 Cost varying fixed charge transportation problem

Suppose there are N -types of vehicles $V_r, r=1, \dots, N$ from each source to each destination. Also it is considered that the fixed charge of N vehicles are F_i^1, \dots, F_i^N for each $i=1, 2, \dots, m$

So, CVFCTP can be represented in the following tabulated form.

	D_1	D_2	..	D_n	stock
O_1	$R_{11}^1 + F_1^1, \dots, R_{11}^N + F_1^N$	$R_{12}^1 + F_1^1, \dots, R_{12}^N + F_1^N$	$R_{1n}^1 + F_1^1, \dots, R_{1n}^N + F_1^N$	a_1
O_2	$R_{21}^1 + F_2^1, \dots, R_{21}^N + F_2^N$	$R_{22}^1 + F_2^1, \dots, R_{22}^N + F_2^N$	$R_{2n}^1 + F_2^1, \dots, R_{2n}^N + F_2^N$	a_2
....
O_m	$R_{m1}^1 + F_m^1, \dots, R_{m1}^N + F_m^N$	$R_{m2}^1 + F_m^1, \dots, R_{m2}^N + F_m^N$	$R_{mn}^1 + F_m^1, \dots, R_{mn}^N + F_m^N$	a_m
Demand	b_1	b_2	b_n	

Table-2: Tabular representation of CVFCTP under N -vehicle

2.4.1 Determination of C_{ij} in CVFCCTP

To solve this problem, apply our proposed algorithms stated as follows:

2.4.2 Algorithm

Step 1. Determine IBFS by any of NWCM, MRMM, MCMM, MMMM, MVAM.

Step 2. After the allocate x_{ij} , determine c_{ij} (unit transportation cost from source O_i to destination D_j) as

$$c_{ij} = \begin{cases} \frac{\sum t_r (R_{ij}^r + F_i^r)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

where t_r are integer solution of

$$\min \sum t_r (R_{ij}^r + F_i^r)$$

$$\text{s.t. } x_{ij} \leq \sum t_r C_r$$

Step 3. For non-basic cell (i, j) possible allocation is the minimum of allocations in i^{th} row and j^{th} column (for possible loop). If possible allocation be x_{ij} , then for non-basic cell c_{ij} (unit transportation cost from source O_i to destination D_j) as :

$$c_{ij} = \begin{cases} \frac{\sum t_r (R_{ij}^r + F_i^r)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

where t_r are integer solution of

$$\min \sum t_r (R_{ij}^r + F_i^r)$$

$$\text{s.t. } x_{ij} \leq \sum t_r C_r$$

In this manner we convert cost varying transportation problem to a usual transportation problem but c_{ij} is not fixed, it may be changed (when this allocation will not serve optimal value) during optimality test.

Step 4. During optimality test some basic cell changes to non-basic cell and some non-basic cell changes to basic cell, depends on running basic cell we first fix c_{ij} by **Step 2** and for non-basic we fix c_{ij} by **Step 3**.

Step 5. Repeat **Step 2.** to **Step 4.** until we obtain optimal solution.

2.4.3 Bi-level Mathematical Programming for CVFCCTP under N -Vehicle

The Bi-level mathematical programming for CVTP under N -vehicle is formulated in **Model 3 (CVFCTPNV)** as follows:

Model 3 (CVFCTPNV)

$$\min \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

where, c_{ij} is determined by following

mathematical programming

$$c_{ij} = \begin{cases} \frac{\sum_r t_r (R_{ij}^r + F_i^r)}{x_{ij}} & \text{if } x_{ij} \neq 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}$$

where $t_r, r = 1, \dots, N$

are integer solution of

$$\min \sum_r t_r (R_{ij}^r + F_i^r)$$

$$s.t. \quad x_{ij} \leq \sum_r t_r C_r$$

$$\sum_{j=1}^n x_{ij} = a_i, \quad i = 1, \dots, m;$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = 1, \dots, n;$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j \quad (6)$$

$$x_{ij} \geq 0 \quad \forall i, \forall j$$

3 Numerical Example

Consider the following FCCTP

	D_1	D_2	D_3	D_4	stock
O_1	0.65	0.74	1.25	2.35	40
O_2	0.56	0.89	2.15	2.34	30
O_3	1.34	0.70	0.45	2.15	50
O_4	3.25	4.21	3.55	1.45	60
Demand	45	25	45	65	

Table : costs of vehicles in a single trip

The route restrictions of this FCCTP are as follows.

$$r_{11} = 12, \quad r_{12} = 15, \quad r_{13} = 17,$$

$$r_{14} = 14 \quad r_{21} = 13, \quad r_{22} = 16, \quad r_{23} = 15, \quad r_{24} = 19$$

$$r_{31} = 11, \quad r_{32} = 22, \quad r_{33} = 18,$$

$$r_{34} = 25 \quad r_{41} = 20, \quad r_{42} = 30, \quad r_{43} = 24, \quad r_{44} = 28$$

Select the vehicles V_1 and V_2 with carrying capacity $c_1 = 7$, $c_2 = 10$. So that rate of restrictions are automatically maintained and considering the transportation cost of vehicles for the a single trip along with their fixed charges for single trip the costs in each cell (i, j)

are represented below: The cost of a single trip of each vehicle in each cell is represented in the following table.

	D ₁	D ₂	D ₃	D ₄	stock
O ₁	10, 13	12, 16	15, 20	8, 12	40
O ₂	20, 25	22, 27	16, 20	12, 17	30
O ₃	6, 9	13, 18	12, 15	10, 14	50
O ₄	15, 19	18, 23	9, 13	14, 18	60
Demand	45	25	45	65	

Table : costs of vehicles in a single trip

The fixed charge of N vehicles are F_i^1, F_i^2 for each $i = 1, 2, 3, 4$ are given bellow.

$$F_1^1 = 2, F_1^2 = 3, F_2^1 = 3, F_2^2 = 4, F_3^1 = 3, F_3^2 = 5, F_4^1 = 4, F_4^2 = 5,$$

Therefore the vehicles V_1 and V_2 with carrying capacity $c_1 = 7$, $c_2 = 10$. So that rate of restrictions are automatically maintained and considering the transportation cost of vehicles for the a single trip along with their fixed charges for single trip the costs in each cell (i, j) are represented below: The cost of a single trip of each vehicle in each cell is represented in the following table.

	D ₁	D ₂	D ₃	D ₄	stock
O ₁	12, 16	14, 19	17, 23	10, 15	40
O ₂	23, 29	25, 31	19, 24	15, 21	30

O ₃	9, 14	16, 23	15, 20	13, 19	50
O ₄	19, 24	22, 28	13, 18	23 60	60
Demand	45	25	45	65	

To solve this problem, the step wise solution procedure is as follows.

Step 1. By MVAM the IBFS is

	D ₁	D ₂	D ₃	D ₄	stock
O ₁	12, 16	x ₁₂ = 5 14, 19	17, 23	x ₁₄ = 35 10, 15	40
O ₂	23, 29	25, 31	19, 24	x ₂₄ = 30 15, 21	30
O ₃	x ₃₁ = 45 9, 14	x ₃₂ = 5 16, 23	15, 20	13, 19	50
O ₄	19, 24	x ₄₂ = 15 22, 28	x ₄₃ = 45 13, 18	23 60	60
Demand	45	25	45	65	

Step 2. Determine c_{ij} by Algorithms TP1

The unit transportation costs in basic cells (by MVAM) are $c_{12} = \frac{14}{5}$, $c_{14} = \frac{55}{35}$,
 $c_{24} = \frac{51}{30}$,

$$c_{31} = \frac{42}{45}, c_{32} = \frac{13}{5}, c_{42} = \frac{41}{15}, c_{43} = \frac{63}{45}$$

and the unit transportation costs in non-basic cells (by MVAM) are $c_{11} = \frac{12}{5}$,
 $c_{13} = \frac{17}{5}$,

$$c_{21} = \frac{87}{30}, c_{22} = \frac{22}{5}, c_{23} = \frac{60}{30}, c_{33} = \frac{12}{5}, c_{34} = \frac{10}{5}, c_{41} = \frac{34}{15}, c_{44} = \frac{32}{15}$$

Step 3. Optimality test:

Determine a set of 8 numbers $u_i, i=1,2,3,4$ and $v_j, j=1,2,3,4$ such that in each cell basic

$$(i, j) \quad c_{ij} = u_i + v_j,$$

each non-basic cell (i, j) by using formula $d_{ij} = c_{ij} - (u_i + v_j)$

So the tabular representation of $u_i, i=1,2,3,4$, $v_j, j=1,2,3,4$ and d_{ij} non-basic cell (i, j)

is given in the following table

	D ₁	D ₂	D ₃	D ₄	u_i
O ₁	$c_{11} = \frac{12}{5}$ 12, 16	$x_{12} = 5$ $c_{12} = \frac{14}{5}$ 14, 19	$c_{13} = \frac{17}{5}$ 17, 23	$x_{14} = 35$ $c_{14} = \frac{55}{35}$ 10, 15	$\frac{14}{5}$
O ₂				$x_{24} = 30$	$\frac{41}{14}$

	$c_{21} = \frac{87}{30}$ 23, 29	$c_{22} = \frac{22}{5}$ 25, 31	$c_{23} = \frac{60}{30}$ 19, 24	$c_{24} = \frac{51}{30}$ 15, 21	
O_3	$x_{31} = 45$ $c_{31} = \frac{42}{45}$ 9, 14	$x_{32} = 5$ $c_{32} = \frac{13}{5}$ 16, 23	$c_{33} = \frac{12}{5}$ 15, 20	$c_{34} = \frac{10}{5}$ 13, 19	$\frac{13}{5}$
O_4	$c_{41} = \frac{34}{15}$ 19, 24	$x_{42} = 15$ $c_{42} = \frac{41}{15}$ 22, 28	$x_{43} = 45$ $c_{43} = \frac{63}{45}$ 13, 18	$c_{44} = \frac{32}{15}$ 23 60	$\frac{41}{15}$
v_j	$\frac{-5}{3}$	0	$\frac{-4}{3}$	$\frac{-43}{35}$	

Since all $d_{ij} > 0$ for all non-basic cell so the table give optimal solution. $x_{12} = 5$, $x_{14} = 35$, $x_{24} = 30$, $x_{31} = 45$, $x_{32} = 5$, $x_{42} = 15$, $x_{43} = 45$, Minimum cost is this stage is $Z^* = 14 + 55 + 51 + 42 + 13 + 41 + 63 = 279$ units(EURO).

4 Conclusions

The proposed method provides a solution for FCTP under finite number of vehicles. Any TP under finite number of vehicles is transform to a CVTP. The IBFS of CVTP are determined

by our modified methods namely, MRMM, MCMM, MMMM and MVAM. Since in CVTP the unit transportation cost is unknown, we determine varying unit transportation cost by our proposed algorithm TP1. To determine the varying unit transportation cost, it is needed to know all information(cost of single trip) of all vehicles in all allocations. Optimality test is nearly similar in a TP. We develop the bi-level mathematical programming not only CVTP but also for CVFCCTP. But further study is needed when the balanced condition (6) is not satisfied.

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